A mathematical approach for the optimal planning of drying operation

Nicolás Vanzetti[1], Gabriela Corsano[1 2], Jorge M. Montagna[1]

[1] Instituto de Desarrollo y Diseño (INGAR, CONICET-UTN), Avellaneda 3657, (S3002GJC) Santa Fe, Argentina.
[2] Facultad Ingeniería Química (FIQ), Universidad Nacional Litoral (UNL) Santiago del Estero 2654, (S3000AOM) Santa Fe, Argentina.
{nvanzetti,gcorsano,montagna}@santafe-conicet.gov.ar

Abstract. Lumber drying is an important stage in sawmills since lumber characteristics required by the market are achieved through this operation. Generally, it is a batch process that is carried out in drying kilns that operate in parallel out of phase. This work presents a mixed integer linear programming model (MILP) for the optimal planning of this process stage. The model determines how board packages are processed in dryers and the start and end times of each drying cycle in each kiln. This approach combines concepts of both multidimensional bin packing –to plan the load of kilns– and parallel machine scheduling –to sequence drying cycles. The model is tested with different objective functions that represent different work criteria related to sawmills.

Keywords: Sawmills, Lumber Dryers, Scheduling

1 Introduction

The forest industry has an important economic, environmental, and social role in the Northeastern region of Argentina (NEA). It involves more than 1000 factories, sawmills making up 95% of those facilities and playing a strategic role in the development of that region.

Among all sawmill operations, lumber drying is an important stage, since it is where the product acquires the characteristics required by markets and its corresponding economical value. Generally, the drying operation is a batch process that is carried out in drying kilns, which can have different dimensions. These kilns are heated using boilers mainly fueled with by-products generated in sawmills (bark, sawdust, chip). The resulting steam heats the air through a heat exchanger, forcing the circulation through the dryers.

Few papers in the literature address the drying operation and most of them are focused either on improving the energy efficiency of the process [1, 2] or on the implementation of new technological alternatives, as is the case of solar dryers [3, 4]. Only Gaudreault et al. [5] plan this operation by developing two models –a mixed integer linear programming (MILP) and a constraint programming model. Both determine the batch to be dried in each kiln and when the drying will be performed, in order to minimize delays in product delivery.

This type of problem can be classified as one of scheduling of non-identical parallel machines and has been addressed by different approaches [6, 7, 8]. In the case of dryers, however, it is not just a matter of sequencing the batches to be dried in each kiln but also of planning how each batch should be loaded.

Planning the filling of dryers may resemble container packaging problems (BPP). This type of problem determines the best way to store objects of different volumes in a finite number of bins, which can be assimilated to loading packages of boards into dryers. Extensive research has dealt with these types of problems [9, 10, 11], but no work was found to combine BPP with scheduling. Therefore, the objective of this work is to generate a new formulation that allows the loading of drying kilns with board packages in order to take advantage of the available space and the corresponding scheduling of the loaded dryers. For this purpose, a MILP model for the optimal planning of drying operation, which combines concepts of BPP and scheduling, is proposed.

2 Problem description

In the following, the operation of dryers in a sawmill is briefly described. Boards are grouped into packages. Each package is made up of boards of a certain family and of the same length. Two boards are assumed to belong to the same family if they have equal thickness, with no additional restriction on width and length.

Packages are mounted on wagons to get into the kilns. All packages loaded on a wagon must be the same length and belong to the same family. Each wagon supports a certain number of packages and, for planning purposes, it is assumed that it adopts the length of packages. Once the wagons are loaded, they enter the dryers via rails and are lined up one behind the other. The dryer is characterized by a number of rails, or available rows, and by its length. Therefore, the number of wagons entering depends on the length of the packages loaded in the wagon and the dimension of the dryer (Fig. 1).

When a kiln is full, a drying cycle begins. The duration of these cycles depends on each dryer size and the characteristics –mainly thickness– of the boards that have been loaded. To guarantee homogeneous drying and better use of resources, all wagons entering the dryer in a given drying cycle must load packages from the same family and must fulfill a certain minimum load, in order to ensure its efficient use. Also, wagons must cover a minimum length per row.

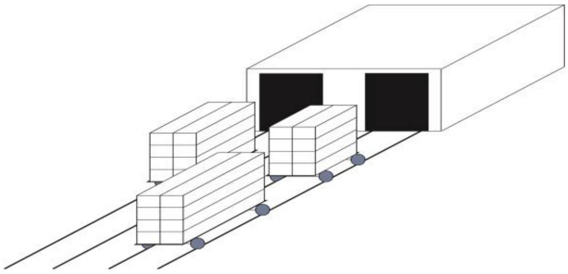

Fig. 1. Entry of wagons to dryers

The drying planning is carried out for a certain time horizon divided into periods, each of them beginning with the entry of new packages from the sawing stage, which will be available to be dried together with packages that have been left over from previous periods. The duration of these periods will vary depending on the frequency of package delivery from sawing stage. Fig. 2 represents a fragment of the time horizon of a drying kiln *d*. $Tmin_t$ and $Tmax_t$ represent the beginning and end of period *t*. TS_{ds} and TF_{ds} determine the start and end point, respectively, of drying cycle *s* in dryer *d*, and $Tdry_{df}$ is the time that required to dry boards of family *f* in dryer *d*. As it can be seen, the duration of each period is not constant since it depends on the time when packages are available. Furthermore, the duration of the drying cycles can vary according to family of packages *f* and the technological characteristics and dimension of dryer *d* (blocks in the Figure).

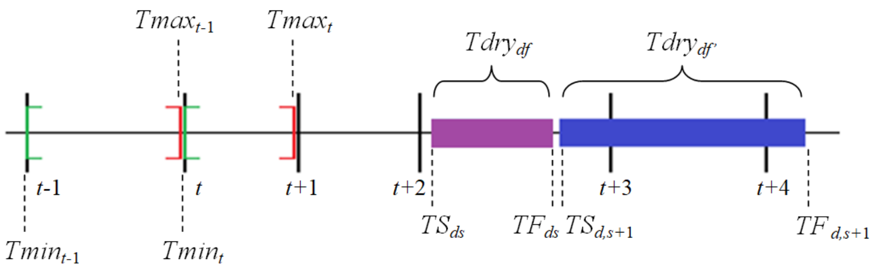

Fig. 2. Timeline for the drying process

The objective of this work is to optimize the drying planning. Basically, board packages that arrive from the sawing stage must be loaded into wagons, wagons must be allocated to dryers, and drying cycles for each kiln must be scheduled. Different criteria for sawmill operation will be analyzed through different objective functions that take into account an efficient use of the dryers, taking advantage of the available time.

3 Model formulation

After the sawing stage, boards belonging to the same family *f* and with the same length *l* are sorted into packages that will be later loaded in wagons *c*. For carrying out the drying stage, there are a number of drying kilns *d*, each one with a certain number of rows $j \in J_d$ where wagons with the same board family are placed. During the planning horizon, each dryer d can perform a certain number of drying cycles $s \in S_d$. These values are proposed by the planner.

Without loss of generality, and in order to eliminate alternative solutions, it is assumed that a set of wagons CD_d is available to be used only in dryer d. Thus, decisions regarding the assignment of a wagon to a dryer are not considered. The maximum number of wagons assigned to the dryer d can be easily determined according to Eq. 1, which considers the length of the dryer $LMax_d$, the shortest length of the packages $Length_l$ and the maximum number of drying cycles allowed along the time horizon.

$$|CD_d| = \frac{LMax_d \, |J_d| \, |S_d|}{\min_{l}\{Length_l\}} \tag{1}$$

Also, the subset CS_{ds} is defined including only the available wagons c for each drying cycle s. In the same way, the subset CJ_{dsj} corresponds to the wagons that are available to be used in row j of drying cycle s of dryer d. The maximum number of wagons that can be assigned to each subset can be easily determined using expressions similar to Eq. (1) from the parameters of the problem. Likewise, the set of wagons available for each drying cycle, CS_s, is established. Therefore, each wagon is univocally assigned to a dryer, a row and a drying cycle, and the model only determines whether or not it is used.

The binary variable z_c determine if the wagon is used during the time horizon,

$$z_c = \begin{cases} 1 \text{ if the wagon } c \text{ is used} \\ 0 \text{ otherwise} \end{cases}$$

In order to define the family and length that are assigned to each wagon, the binary variables r_{cf} and p_{cl}, are stated.

$$r_{cf} = \begin{cases} 1 \text{ if the wagon } c \text{ is used with packages of family } f \\ 0 \text{ otherwise} \end{cases}$$

$$p_{cl} = \begin{cases} 1 \text{ if the wagon } c \text{ is used with packages of length } l \\ 0 \text{ otherwise} \end{cases}$$

Eq. 2 states that each wagon is loaded with boards of a unique family of packages, while Eq. 3 allows only one length, if the wagon is used:

$$z_c = \sum_{f} r_{cf} \qquad \forall\, c \tag{2}$$

$$z_c = \sum_{l} p_{cl} \qquad \forall\, c \tag{3}$$

With the aim of simultaneously relating the family and the length, the continuous variable h_{cfl} is used. Eq. 4 and 5 define the variable h_{cfl} equal to 1 only if r_{cf} and p_{cl} are 1:

$$h_{cfl} \geq r_{cf} + p_{cl} - 1 \qquad \forall\, c, f, l \tag{4}$$

$$h_{cfl} \leq r_{cf} \qquad \forall\, c, f, l \quad ; \quad h_{cfl} \leq p_{cl} \qquad \forall\, c, f, l \tag{5}$$

Eq. 6 forces that the variable h_{cfl} be positive if the wagon is used.

$$z_c = \sum_l \sum_f h_{cfl} \qquad \forall\, c \tag{6}$$

It is worth to note that according to Eqs. 2 and 3, z_c can be defined as continuous variable in [0, 1], as well as h_{cfl} through Eqs. 4-6.

Throughout the studied time horizon, NP_{flt} new packages of boards of the family f and length l enter the drying stage in period t. These packages are added to those that are part of the inventory of packages with the same characteristics of the previous period ($I_{fl,t-1}$). As previously stated, the planning horizon is divided into time periods t, which start at the moment that packages arrive at this stage and are available to be dried. The duration of these periods depends on the frequency in which new packages arrive, which may or may not be of the same duration.

From the total packages available in period t, some of them are assigned to wagons c to be dried in different kilns d, and the rest are stored for future periods (I_{flt}). Therefore, the number of packages in stock at the end of the previous period plus the amount of packages that enter at the beginning of the period must be equal to the number of packages that enters to the drying kilns plus those that are kept in inventory for the following periods (Eq. 7). Qp_{cflt} represents the number of packages of family f and length l that are assigned to wagon c used in time period t.

$$I_{fl,t-1} + NP_{flt} = I_{flt} + \sum_d \sum_s \sum_{c \in CS_{ds}} Qp_{cflt} \qquad \forall f, l, t \tag{7}$$

Eq. 8 states that the number of packages that a wagon can load is between a minimum value ($CMin_d$) and a maximum value ($CMax_d$) established by productivity conditions determined by the firm for each dryer and its capacity.

$$CMin_d\, h_{cfl} \leq \sum_t Qp_{cflt} \leq CMax_d h_{cfl} \quad \forall\, d, c \in CS_{ds}, f, l, s \in S_d \tag{8}$$

The binary variable i_{ct} is defined for determining the time period t that a certain wagon c is used:

$$i_{ct} = \begin{cases} 1 \text{ if the wagon } c \text{ is used in the time period } t \\ 0 \text{ otherwise} \end{cases}$$

Eq. 9 guarantees that the each wagon is only used in a period of time, if it is employed:

$$z_c = \sum_t i_{ct} \qquad \forall\, c \tag{9}$$

Eq. 10 ensures that Qp_{cflt} is zero if the wagon c is not selected in time period t.

$$Qp_{cflt} \leq CMax_d\, i_{ct} \qquad \forall\, d, c \in CS_{ds}, l, f, t, s \in S_d \tag{10}$$

The length occupied by the wagons in each row of a dryer in each drying cycle cannot exceed the length of the dryer ($LMax_d$) and must be greater than a minimum length ($LMin_d$) that the company establishes to ensure a good utilization of the dryer (Eq. 11). The inner summation of Eq. 11 only considers wagons that are simultaneously assigned to a certain row of a dryer, during a determined drying cycle.

$$LMin_d\, x_{ds} \leq \sum_l \sum_{c \in CJ_{dsj}} Length_l\, p_{cl} \leq LMax_d\, x_{ds} \qquad \forall\, d, f, j \in J_d, s \in S_d \tag{11}$$

where x_{ds} is a binary variable whose value is:

$$x_{ds} = \begin{cases} 1 \text{ if the drying cycle } s \text{ is used in the dryer } d \\ 0 \text{ otherwise} \end{cases}$$

It is worth to highlight that the bounds established by Eqs. 8-10 are stated in order to improve the dryer performance and the packages management. These conditions allow configuring alternatives for the occupation of the dryers according to the policies that the company wants to implement related to the productivity of the dryers. These conditions are easy to comply when board production levels are high, but difficult to reach when the production level is low.

In this approach, two temporal aspects are managed: the time period t that indicates the when the new boards packages arrive to be dried, and the drying cycle s that indicates when the drying operation begins.

In order to determine the period t in which a drying cycle s starts, the binary variable u_{dst} is defined:

$$u_{dts} = \begin{cases} 1 \text{ if the dryer } d \text{ begins its drying cycle } s \text{ in period } t \\ 0 \text{ otherwise} \end{cases}$$

Eq. 12 allows relating the beginning of the drying cycle s with the corresponding period t.

$$\sum_t (Tmin_t)\, u_{dst} \leq TS_{ds} \leq \sum_t (Tmax_t)\, u_{dst} \qquad \forall\, d, s \in S_d \tag{12}$$

where $Tmin_t$ and $Tmax_t$ are model data and represent the beginning and end of each period of time, respectively.

Eq. 13 determines the final time for each drying cycle s in each dryer d, TF_{ds}.

$$TF_{ds} = TS_{ds} + \sum_{f} Tdry_{df}\, v_{dfs} \qquad \forall d, s \in S_d \tag{13}$$

TS_{ds} determines the starting point of each drying cycle s in a dryer d, while the parameter $Tdry_{df}$ is the duration of each cycle, which depends on the size of the drying kiln d and the board family f to be dried in it. The binary variable v_{dfs} is defined as:

$$v_{dfs} = \begin{cases} 1 \text{ if dryer } d \text{ is filled with packages of family } f \text{ in its drying cycle } s \\ 0 \text{ otherwise} \end{cases}$$

Eq. 14 and 15 avoid the overlapping of drying cycles in the same dryer, where N is a sufficient large constant that can be calculated as the parameter $Tmax_t$ for the last period (t = last) plus the maximum processing time:

$$N = \max_{t=last}\{Tmax_t\} + \max_{d,f}\{Tdry_{df}\}$$

$$TF_{ds} - TS_{d,s+1} \le N\left(1 - x_{d,s+1}\right) \qquad \forall d, s < |S_d| \tag{14}$$

$$TS_{ds} \le N\, x_{ds} \qquad \forall d, s \in S_d \tag{15}$$

The following equations relate the variables x_{ds}, v_{dfs} and u_{dst}. Eq. 16 state that during a drying cycle s only packages of one family f can be dried, while Eq. 17 indicate that only one drying cycle begins in a time period t.

$$x_{ds} = \sum_{f} v_{dfs} \qquad \forall\, d, s \in S_d \tag{16}$$

$$x_{ds} = \sum_{t} u_{dst} \qquad \forall\, d, s \in S_d \tag{17}$$

In order to avoid symmetric solutions, the following constraints are added. Eq. 18 states that wagons are assigned in ascending order.

$$z_{c+1} \le z_c \qquad \forall\, c \in CJ_{dsj} \tag{18}$$

Eq. 19 sorts the length of the wagons in each row of a dryer from the highest to the lowest.

$$\sum_{l} Length_l\; p_{cl} \ge \sum_{l} Length_l\, p_{c+1,l} \quad \forall\, d, c \in CJ_{dsj}, j \in J_d, s \in S_d \tag{19}$$

Eq. 20 states that for each used dryer, the occupied length of a row must be less than or equal to that of the previous row.

$$\sum_{l} \sum_{c \in CJ_{dsj}} Length_l\, p_{cl} \ge \sum_{l} \sum_{c \in CJ_{dsj+1}} Length_l\, p_{cl} \quad \forall\, d, j \in J_d, s \in S_d \tag{20}$$

Drying cycles are also used in ascending order (Eq. 21):

$$x_{d,s+1} \leq x_{ds} \qquad \forall d, s \in S_d \tag{21}$$

The different binary variables are related through logical relationships. The following constraints establish these relations in order to reduce the search space and improve the model performance.

Eqs. 22 and 23 allow the use of wagons in a dryer *d* only if it performs a drying cycle *s*.

$$x_{ds} \leq \sum_{c \in CS_{ds}} \sum_{l} \sum_{f} h_{cfl} \qquad \forall\, d, s \in S_d \tag{22}$$

$$x_{ds} \geq h_{cfl} \qquad \forall\, d, c \in CS_{ds}, l, f, s \in S_d \tag{23}$$

Eqs. 24 and 25 ensure that h_{cfl} and r_{cf} are zero if there are no packages of family *f* assigned to dryer *d* in cycle *s*.

$$\sum_{c \in CS_{ds}} \sum_{l} h_{cfl} \leq |J_d| M\, v_{dfs} \qquad \forall\, d, f, s \in S_d \tag{24}$$

$$\sum_{c \in CS_{ds}} r_{cf} \leq |J_d| M\, v_{dfs} \qquad \forall\, d, s \in S_d, t \tag{25}$$

where *M* is a parameter equal to the largest number of wagons that can be accommodated in a row: $M = \frac{\max_{d}\{Lmax_d\}}{\min_{l}\{Length_l\}}$.

Eq. 26 ensures that no wagons are assigned to a certain dryer *d* to be processed in period t (i_{ct} =0) if such dryer is not used during a drying cycle *s* in period *t*. Otherwise, the total wagons assigned to the dryer is upper bounded.

$$|J_d| M\, u_{dst} \geq \sum_{c \in CS_{ds}} i_{ct} \qquad \forall\, d, s \in S_d, t \tag{26}$$

Eqs. 27 and 28 determine that if no wagons of family *f* and length *l* are assigned to dryer *d* during drying cycle *s*, then that dryer is not used in that cycle, otherwise the total number of wagons in the dryer must be less than maximum number of wagons that can be accommodated in a row by the number of row in the dryer.

$$\sum_{t} u_{dst} \leq \sum_{c \in CD_d \cap CS_s} \sum_{f} \sum_{l} h_{cfl} \qquad \forall\, d, s \in S_d \tag{27}$$

$$|J_d| M \sum_{t} u_{dst} \geq \sum_{c \in CS_{ds}} \sum_{f} \sum_{l} h_{cfl} \qquad \forall\, d, s \in S_d \tag{28}$$

The operating criteria in the drying stage can vary according to different reasons: the production level and market requirements, the availability of resources, etc. These criteria can be contrasted between them, and this analysis is really interesting. Next, the objectives posed by each of them are detailed.

Objective function 1 (FO 1): Maximize the number of dried packages.

This criterion seeks to dry as many packets of boards as possible in the defined time horizon (Eq. 29). It is a very appropriate measure, although it can generate inefficient use of kilns.

$$\max FO1 = \sum_{c}\sum_{l}\sum_{f}\sum_{t} Qp_{cflt} \tag{29}$$

Objective function 2 (FO 2): Minimize non-occupied drying

This criterion aims at a more efficient use of the capacity of the dryers (Eq. 30).

$$\min FO2 = \sum_{d}\sum_{s}\left(x_{ds} - \sum_{c \in CS_{ds}}\sum_{l}\sum_{f}\sum_{t} \frac{Qp_{clft}\ Area\ Length_l}{VolDry_d} \right) \tag{30}$$

where *Area* corresponds to the transversal area of the packages, the same for all of them in the wagon *c*. $VolDry_d$ is the volume of the drying kiln *d*.

Also, an upper bound for the packages inventory at the end of the time horizon is required, which is calculated as a percentage (*fp*) of the total number of packages along the time horizon (Eq. 31).In this way, a minimum number of dried packages are ensured.

$$\sum_{f}\sum_{l} I_{fl,t=last} \leq fp \sum_{f}\sum_{l}\left(I_{fl,t=0} + \sum_{t} NP_{flt} \right) \tag{31}$$

4 Examples

To check the performance of the model, the case study corresponds to a sawmill with 4 drying kilns. Dryers are of different sizes and accept different amounts of packages per wagon (Table 1). The drying kiln has two rows for the entrance of wagons and 5 drying cycles can be carried out in each one. Also, not all of them are available for use at the beginning of the time horizon, as they are busy with a previous drying plan.

Table 1. Dryer capacity

Dryer	Length (feet)		Packages by wagons (U)	
	LMin	***LMax***	***CMin***	***CMax***
d_1-d_2	23	30	8	10
d_3-d_4	30	40	12	14

The adopted horizon is equal to 5 days and it is considered that at the beginning of the planning horizon there are no packages in inventory ($I_{fl,t=0}$=0). In addition, the final inventory of not dried packages is limited to 25% of the total produced packages. The packages are classified into 3 families of thicknesses with 6 possible lengths. The total packages to be dried are 697, which enter this stage as a result of sawing production in the periods indicated in Table 2. Due to the lack of space, the remaining data used to solve this example is not shown but is available to anyone who requests it.

Table 2. Number and family of packages that enter for each period

Family	Length	Time Period				
		t_1	t_2	t_3	t_4	t_5
f_1	l_1	34	0	9	20	0
	l_2	0	24	12	0	0
	l_3	22	0	10	0	8
	l_4	0	24	0	21	0
	l_5	0	26	0	0	14
	l_6	36	0	0	13	0
f_2	l_1	26	0	15	14	0
	l_2	0	20	10	0	0
	l_3	26	0	18	0	12
	l_4	0	18	0	23	0
	l_5	0	15	0	0	9
	l_6	16	0	0	25	0
f_3	l_1	10	0	10	17	0
	l_2	0	14	20	0	0
	l_3	10	0	23	0	16
	l_4	0	9	0	8	0

I_5	0	5	0	0	9
I_6	10	0	0	16	0

The example is implemented and solved in GAMS [12] using CPLEX 12.6 solver in an Intel(R) Core(TM) i7-3770, 3.40 GHz. The model has 29228 equations, 14575 continuous variables and 5460 binary variables.

Table 3 shows the most outstanding results of the two objective functions.

Table 3. Results of Example

	FO 1	FO 2
Dried packages (u)	**697**	**526**
Drying Cycles	13	8
Unused volume (%)	15.9	0
Final inventory (u)	0	171
Inventory (%)	0	24.5
CPU Time (sec)	727	489
% GAP	0	0

With FO 1, 13 drying cycles are used throughout the planning horizon, drying all available packages, leaving no packages in inventory. In this case, when as many packages as possible are dried without considering the efficiency of the dryers (only a lower bound for the occupation is required), the unused volume of the dryers is the highest (15.9%).

Using FO 2, 524 packages are dried in 8 drying cycles. In each cycle, the entire available volume is used without leaving free space. In contrast to FO 1, the final inventory is 24.5% of the total packages, just below the allowed limit (25%).

Fig. 3 shows the Gantt chart, depicting in red the drying cycles performed for packages of family 1, in green those of family 2, and in blue the corresponding to family 3. In gray, the drying cycle from a previous period is shown. The vertical lines symbolize the beginning of each period, or in other words, the entry of new packages from the sawing sector to be dried.

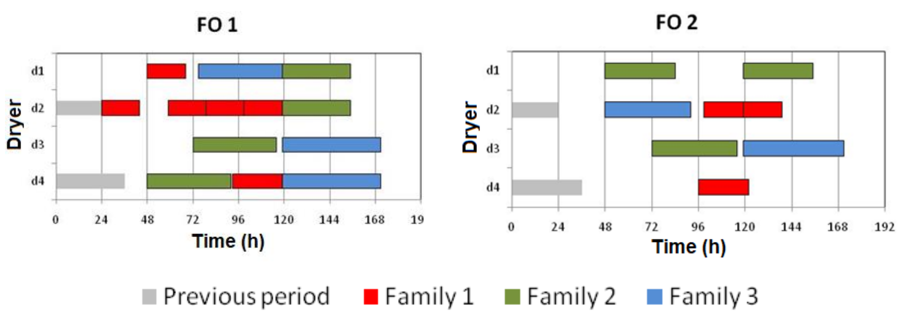

Fig. 3. Gantt chart of the drying sequence

From Fig. 3, it can be seen that for FO 2 the drying cycles begin when there are enough packages to complete the volume of the dryer, while, for FO 1, they start when there is an adequate number of packages to carry out the process. Also, it is observed that the inactivity times are shorter for FO 1 with respect to FO2, due to the fact that more drying cycles are carried out.

More detailed analysis can be made to show how the operation of the drying sector varies according to the adopted criterion. The proposed formulation is a powerful tool for managing this sector, in order to compare different solutions. Obviously the best approach will depend on the particular context of sawmill operations at the planning time.

Tables 4 show how the dryer d_1 is filled in the different drying cycles for FO 1. A similar table is obtained for each of the dryers. In the table, the package family dried in each cycle, the length adopted by each wagon and the number of packages loaded in each wagon, are displayed. The length covered in each row and the percentage of use of each kiln per cycle, is also depicted. As shown in the tables and graphs, the model provides a detailed description of how each kiln must be loaded in each drying cycle. This is a very valuable result for the operation of this sector of the plant.

Table 4. Distribution of packages in drying kilns for FO 1

Cycle	Family	Row	ID wagon - length - Number of packages per wagon			Used Length	Occupied Per. (%)
s_1	f_1	j_1	c_1-l_2-10	c_2-l_2-8	c_3-l_2-10	30	90.7
		j_2	c_4-l_6-8	c_5-l_3-10		30	
s_2	f_3	j_1	c_7-l_4-8	c_8-l_3-10		26	76.3
		j_2	c_{10}-l_4-9	c_{11}-l_2-10		24	
s_3	f_2	j_1	c_{13}-l_5-10	c_{14}-l_4-9		30	89.7
		j_2	c_{16}-l_4-10	c_{17}-l_4-8		28	

5 Conclusions

In this work, an MILP model was presented for the multiperiod planning of the wood drying stage of a sawmill considering different families and lengths of boards, and drying kilns of different dimensions. This formulation allows determining which family of packages is dried in each dryer, the assignment of packages to wagons and wagons to dryer rows in each drying cycle, initial and final processing times for each cycle, the final inventory and the used capacity of the dryers.

Two objective functions were presented, which correspond to possible operating criteria. As expected, the results obtained by the objective functions generate opposite drying plans, on the one hand the level of production is emphasized, and on the other the efficiency in the use of resources. The presented model is a useful tool for sawmills allowing an efficient use of available resources.

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Nomenclature

Sets

D	Dryers
F	Families of packages
J_d	Rows for dryer *d*
L	Length of packages
CD_d	Wagons assigned to dryer *d*
CS_s	Wagons assigned to drying cycle *s*
CJ_j	Wagons assigned to row *j*
S_d	Drying cycles allowed for dryer *d*
T	Time periods

Parameters

$CMax_d$	Maximum number of packages for a wagon assigned to the dryer *d*
$CMin_d$	Minimum number of packages for a wagon assigned to the dryer *d*
$Length_l$	Length of packages
$LMax_d$	Maximum length of dryer *d*
$LMin_d$	Minimum length to use of dryer *d*
M	Big constant
N	Big constant
NP_{flt}	Number of available packages belonging to family *f* and length *l* in period *t*
$Tdry_{df}$	Processing time for packages of family *f* in dryer *d*
$TMax_t$	Final time of period *t*
$TMin_t$	Initial time of period *t*

Binary variables

i_{ct}	Indicates if wagon *c* is used in time period *t*
p_{cl}	Indicates if wagon *c* is used with packages of length *l*
r_{cf}	Indicates whether wagon *c* is used with packages of family *f*
u_{dst}	Indicates if drying cycle *s* of dryer *d* begins in period of time *t*
v_{dfs}	Indicates if packages of family *f* enter to dryer *d* in the drying cycle *s*
x_{ds}	Indicates if dryer *d* used drying cycle *s*
z_c	Indicates if the wagon *c* is used

Continuous variables

h_{cfl}	Indicates if wagon *c* is used with packages of family *f* and length *l*
I_{flt}	Inventory of packages of family *f* and length *l* in the time period *t*
Qp_{cflt}	Amount of packages of family *f* and length *l* that are loaded in wagon *c* and dried in time period *t*
TF_{ds}	Final time of drying cycle *s* in dryer *d*
TS_{ds}	Initial time of drying cycle *s* in dryer *d*